

Suppression of Weiss oscillations in the magnetoconductivity of modulated graphene monolayer

M. Tahir*

Department of Physics, University of Sargodha, Sargodha 40100, Pakistan

K. Sabeeh†

Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan

(Date textdate; Received textdate; Revised textdate; Accepted textdate; Published textdate)

Abstract

We have investigated the electrical transport properties of Dirac electrons in a monolayer graphene sheet in the presence of both electric and magnetic modulations. The effects of the modulations on quantum transport when they are in phase and out of phase are considered. We present the energy spectrum and the bandwidth of the Dirac electrons in the presence of both the modulations. We determine the σ_{yy} component of the magnetoconductivity tensor for this system which is shown to exhibit Weiss oscillations. Asymptotic expressions for σ_{yy} are also calculated to better illustrate the effects of in-phase and out-of-phase modulations. We find that the position of the oscillations in magnetoconductivity depends on the relative strength of the two modulations. When the two modulations are out-of-phase there is complete suppression of Weiss oscillations for particular relative strength of the modulations.

I. INTRODUCTION

The recent successful preparation of monolayer graphene has generated a lot of interest in the physics community and efforts are underway to study the electronic properties of graphene [1]. The nature of quasiparticles called Dirac electrons in these two-dimensional systems is very different from those of the conventional two-dimensional electron gas (2DEG) realized in semiconductor heterostructures. Graphene has a honeycomb lattice of carbon atoms. The quasiparticles in graphene have a band structure in which electron and hole bands touch at two points in the Brillouin zone. At these Dirac points the quasiparticles obey the massless Dirac equation. In other words, they behave as massless Dirac particles leading to a linear dispersion relation $\epsilon_k = vk$ (with the characteristic velocity $v \simeq 10^6 m/s$). This difference in the nature of the quasiparticles in graphene from conventional 2DEG has given rise to a host of new and unusual phenomena such as anomalous quantum Hall effects and a π Berry phase[1][2]. Among the electronic properties of interest is the interaction of electrons with artificially created periodic potentials. It has been observed that if conventional 2DEG is subjected to artificially created periodic potentials in the submicrometer range it leads to the appearance of Weiss oscillations in the magnetoresistance. This type of electrical modulation of the 2D system can be carried out by depositing an array of parallel metallic strips on the surface or through two interfering laser beams [3, 4, 5]. Besides the fundamental interest in understanding the electronic properties of graphene there is also serious suggestions that it can serve as the building block for nanoelectronic devices [6].

In conventional 2DEG systems, electron transport in the presence of electric and magnetic modulation has continued to be an active area of research [7, 8, 9]. Recently, electrical transport in graphene monolayer in the presence of electrical modulation was considered and theoretical predictions made [10]. We have also carried out a study of magnetoconductivity when graphene monolayer is subjected to magnetic modulation alone[11]. Along the same lines, in this work we investigate low temperature magnetotransport of Dirac electrons in a graphene monolayer subjected to both electric and magnetic modulations with the same period. From a practical point of view, this is important to consider as magnetic modulation of graphene can be realized by magnetic or superconducting stripes placed on top of graphene which in turn act as electrical gates that induce electric modulation. The relative phase of the two modulations can have important consequences for magnetotransport in the system

as was seen in conventional 2DEG. Therefore in this work we investigate Weiss oscillations in magnetoconductivity σ_{yy} in a graphene monolayer for both the cases when the electric and magnetic modulations are in-phase and when they are out of phase.

II. ENERGY SPECTRUM AND BANDWIDTH

We consider two-dimensional Dirac electrons in graphene moving in the $x - y$ -plane. The magnetic field (B) is applied along the z -direction perpendicular to the graphene plane. We consider the perpendicular magnetic field B modulated weakly and periodically along one direction such that $\mathbf{B} = (B + B_0 \cos(Kx))\mathbf{z}$. Here B_0 is the strength of the magnetic modulation and $K = 2\pi/a$ with a being the modulation period. We consider the modulation to be weak such that $B_0 \ll B$. We use the Landau gauge for vector potential $\mathbf{A} = (0, Bx, 0)$. In effective mass approximation the one electron Hamiltonian is $H = v\overleftrightarrow{\sigma} \cdot (-i\hbar\nabla + e\mathbf{A})$. The low energy excitations are described by the two-dimensional (2D) Dirac like Hamiltonian [1, 2, 10] $H = v\overleftrightarrow{\sigma} \cdot (-i\hbar\nabla + e\mathbf{A})$. Here $\overleftrightarrow{\sigma} = \{\overleftrightarrow{\sigma}_x, \overleftrightarrow{\sigma}_y\}$ are the Pauli matrices and v characterizes the electron velocity. We employ the Landau gauge and write the vector potential as $\mathbf{A} = (0, Bx + (B_0/K) \sin(Kx), 0)$. The Hamiltonian can be written as $H = H_0 + H'$ where H_0 is the unmodulated Hamiltonian given as $H_0 = -i\hbar v\overleftrightarrow{\sigma} \cdot \nabla + ev\overleftrightarrow{\sigma}_y Bx$ and $H' = ev\overleftrightarrow{\sigma}_y \frac{B_0}{K} \sin(Kx)$. The Landau level energy eigenvalues without modulation are given by

$$\varepsilon(n) = \hbar\omega_g\sqrt{n} \quad (1)$$

where n is an integer and $\omega_g = v\sqrt{2eB/\hbar}$. As has been pointed out [10] the Landau level spectrum for Dirac electrons is significantly different from the spectrum for electrons in conventional 2DEG which is given as $\varepsilon(n) = \hbar\omega_c(n+1/2)$, where $\omega_c = eB/m$ is the cyclotron frequency.

The eigenfunctions without modulation are given by

$$\Psi_{n,k_y}(r) = \frac{e^{ik_y y}}{\sqrt{2L_y}} \begin{pmatrix} -i\varphi_{n-1}(x, x_0) \\ \varphi_n(x, x_0) \end{pmatrix} \quad (2)$$

where $\varphi_n(x, x_0) = \frac{e^{-(x+x_0)/2l}}{\sqrt{2^n n! l \sqrt{\pi}}} H_n\left(\frac{x+x_0}{l}\right)$ are the normalized harmonic oscillator eigenfunctions, $l = \sqrt{\hbar/eB}$ is the magnetic length, $x_0 = l^2 k_y$, L_y is the y -dimension of the graphene layer and $H_n(x)$ are the Hermite polynomials.

Since we are considering weak modulation $B_0 \ll B$, we can apply standard perturbation theory to determine the first order corrections to the unmodulated energy eigenvalues in the presence of modulation with the result $\Delta\varepsilon_{n,k_y} = \omega_0 \cos(Kx_0) \left(\sqrt{\frac{2n}{u}} e^{-u/2} [L_{n-1}(u) - L_n(u)] \right)$ where $\omega_0 = \frac{evB_0}{K}$, $u = K^2 l^2 / 2$ and $L_n(u)$ are the Laguerre polynomials. Hence the energy eigenvalues in the presence of the periodic magnetic modulation are

$$\varepsilon(n, x_0) = \varepsilon(n) + \Delta\varepsilon_{n,k_y} = \hbar\omega_g \sqrt{n} + \omega_0 \cos(Kx_0) G_n \quad (3)$$

with $G_n(u) = \sqrt{\frac{2n}{u}} e^{-u/2} [L_{n-1}(u) - L_n(u)]$. We observe that the degeneracy of the Landau level spectrum of the unmodulated system with respect to k_y is lifted in the presence of modulation with the explicit presence of k_y in x_0 . The $n = 0$ Landau level is different from the rest as the energy of this level is zero and electrons in this level do not contribute to diffusive conductivity calculated in the next section. The rest of the Landau levels broaden into bands. The Landau bandwidths $\sim G_n$ oscillates as a function of n since $L_n(u)$ are oscillatory functions of the index n .

Since we are interested in electron transport in the presence of both electric and magnetic modulations, we consider an additional weak electric modulation potential given as $V(x) = V_0 \cos(Kx)$ on the system. Here V_0 is the amplitude of modulation. We can determine the energy eigenvalues in the presence of weak electric modulation where we take V_0 to be an order of magnitude smaller than the Fermi Energy $\varepsilon_F = v_F \hbar k_F$ with $k_F = \sqrt{2\pi n_e}$ is the magnitude of Fermi wave vector with n_e being the electron concentration. Hence we can apply standard first order perturbation theory to determine the energy eigenvalues in the presence of modulation. The first order correction in the energy eigenvalues when electric modulation is present is given as

$$\varepsilon(n, x_0) = \varepsilon(n) + V_0 F_n \cos(Kx_0) \quad (4)$$

Here, $F_n = \frac{1}{2} \exp(-\frac{u}{2}) [L_n(u) + L_{n-1}(u)]$, $u = \frac{K^2 l^2}{2}$ and, $L_n(u)$ and $L_{n-1}(u)$ are Laguerre polynomials.

The energy eigenvalues of Dirac electrons in the presence of both modulations can be expressed as

$$\varepsilon(n, k_y) = \hbar\omega_g \sqrt{n} + \omega_0 \cos(Kx_0) G_n + V_0 F_n \cos(Kx_0). \quad (5)$$

To better appreciate the modulation effects on the Landau levels we determine the asymptotic expression for the bandwidth (Δ) next. The width of the n th Landau level in the

presence of periodic electric and magnetic modulation is given as $\Delta = \Delta_B + \Delta_E$, where Δ_E is width of the electric modulation and Δ_B is the width of the magnetic modulation:

$$\Delta_B = 2 |G_n| = 2 \sqrt{\frac{2n}{u}} e^{-u/2} [L_{n-1}(u) - L_n(u)] \quad (6)$$

The asymptotic expression of bandwidth can be obtained by using the following asymptotic expression for the Laguerre polynomials by taking the large n ($n_F = \frac{\varepsilon_F^2}{\varepsilon^2(n)} = \frac{\hbar K_F^2}{2eB}$) limit as

$$\exp^{-u/2} L_n(u) \rightarrow \frac{1}{\sqrt{\pi \sqrt{nu}}} \cos(2\sqrt{nu} - \frac{\pi}{4})$$

with the result that the asymptotic expression for Δ_B is

$$\Delta_B = \frac{8\omega_0}{Kl} \sqrt{\frac{\hbar K_F^2}{2eB\pi^2 R_g}} \sin\left(\frac{2eB\pi R_g}{2a\hbar K_F^2}\right) \sin\left(\frac{2\pi R_g}{a} - \frac{\pi}{4}\right). \quad (7)$$

Similarly, for electric modulation, the bandwidth Δ_E is given as

$$\Delta_E = 2 |F_N| = V_0 \exp^{-\frac{u}{2}} |L_n(u) + L_{n-1}(u)|.$$

and the asymptotic expression for Δ_E is

$$\Delta_E = V_0 \left(\frac{a}{\pi^2 R_g}\right)^{\frac{1}{2}} \cos\left(\frac{2eB\pi R_g}{2a\hbar K_F^2}\right) \cos\left(\frac{2\pi R_g}{a} - \frac{\pi}{4}\right). \quad (8)$$

Therefore the bandwidth in the presence of both electric and magnetic modulations can be expressed as

$$\Delta = \frac{8\omega_0}{Kl} \sqrt{\frac{\hbar K_F^2}{2eB\pi^2 R_g}} \sin\left(\frac{2eB\pi R_g}{2a\hbar K_F^2}\right) \sin\left(\frac{2\pi R_g}{a} - \frac{\pi}{4}\right) + V_0 \left(\frac{a}{\pi^2 R_g}\right)^{\frac{1}{2}} \cos\left(\frac{2eB\pi R_g}{2a\hbar K_F^2}\right) \cos\left(\frac{2\pi R_g}{a} - \frac{\pi}{4}\right). \quad (9)$$

In Fig.(1) we present the bandwidths as a function of the magnetic field for temperature $T = 6K$, electron density $n_e = 3 \times 10^{11} cm^{-2}$, the period of modulation $a = 350nm$. The strength of the electric modulation $V_0 = 0.2meV$ where as $B_0 = 0.004T$ which corresponds to $\omega_0 = 0.2meV$ with the result that both the modulations have equal strengths. In the same figure we have also shown the bandwidths when either the magnetic or electric modulation alone is present. We observe that electric and magnetic bandwidths are out of phase while the positions of the extrema of combined bandwidth are shifted with respect to electric and magnetic bandwidths. The combined bandwidth when both modulations are present will affect the conductivity and that is considered in the next section.

III. MAGNETOCONDUCTIVITY WITH PERIODIC ELECTRIC AND MAGNETIC MODULATION: IN-PHASE

To calculate the electrical conductivity in the presence of weak electric and magnetic modulations we use Kubo formula to calculate the linear response to applied external fields. In a magnetic field, the main contribution to Weiss oscillations comes from the scattering induced migration of the Larmor circle center. This is diffusive conductivity and we shall determine it following the approach in [7, 10, 11, 12] where it was shown that the diagonal component of conductivity σ_{yy} can be calculated by the following expression in the case of quasielastic scattering of electrons

$$\sigma_{yy} = \frac{\beta e^2}{L_x L_y} \sum_{\zeta} f(E_{\zeta}) [1 - f(E_{\zeta})] \tau(E_{\zeta}) (v_y^{\zeta})^2 \quad (10)$$

L_x , L_y , are the dimensions of the layer, $\beta = \frac{1}{k_B T}$ is the inverse temperature with k_B the Boltzmann constant, $f(E)$ is the Fermi Dirac distribution function, $\tau(E)$ is the electron relaxation time and ζ denotes the quantum numbers of the electron eigenstate. The diagonal component of the conductivity σ_{yy} is due to modulation induced broadening of Landau bands and hence it carries the effects of modulation in which we are primarily interested in this work. σ_{xx} does not contribute as the component of velocity in the x -direction is zero here. The collisional contribution due to impurities is not taken into account in this work.

The summation in Eq.(10) over the quantum numbers ζ can be written as

$$\frac{1}{L_x L_y} \sum_{\zeta} = \frac{1}{2\pi L_x} \int_0^{\frac{L_x}{l^2}} dk_y \sum_{n=0}^{\infty} = \frac{1}{2\pi l^2} \sum_{n=0}^{\infty} \quad (11)$$

The component of velocity required in Eq.(10) can be calculated from the following expression

$$v_y^{\zeta} = -\frac{\partial}{\hbar \partial k_y} \varepsilon(n, k_y). \quad (12)$$

Substituting the expression for $\varepsilon(n, k_y)$ obtained in Eq.(5) into Eq.(12) yields

$$v_y^{\zeta} = \left[\frac{2\omega_0 u}{\hbar K} \sin(Kx_0) G_n(u) + \frac{2V_0 u}{\hbar K} \sin(Kx_0) F_n(u) \right] \quad (13)$$

As a result v_y^{ζ} , the corresponding velocity given by Eq.(13) contains the contribution from both the modulations (electric and magnetic) obtained in Eq.(5) compared to one term in

velocity component for each[10, 11]. This term (v_y^s) has important consequences for the quantum transport phenomena in modulated systems.

With the results obtained in Eqs.(11), (12) and (13) we can express the diffusive contribution to the conductivity given by Eq.(10) as

$$\sigma_{yy} = A_0 \phi \quad (14)$$

where

$$A_0 = \frac{e^2 \tau \beta}{\pi \hbar^2} \quad (15)$$

and ϕ is given as

$$\phi = \sum_{n=0}^{\infty} \frac{g(\varepsilon_n)}{[g(\varepsilon_n) + 1]^2} \left[\frac{\sqrt{u} e^{-u/2} V_0}{2} (L_n(u) + L_{n-1}(u)) + \omega_0 e^{-u/2} \sqrt{2n} (L_{n-1}(u) - L_n(u)) \right]^2 \quad (16)$$

where $g(\varepsilon) = \exp[\beta(\varepsilon - \varepsilon_F)]$ and ε_F is the Fermi energy.

In Fig.(2) we show the in-phase conductivity (the magnetic and the electric modulations are in-phase) σ_{yy} given by Eqs(14,15,16) as a function of the inverse magnetic field for temperature $T = 6K$, electron density $n_e = 3 \times 10^{11} cm^{-2}$, the period of modulation $a = 350nm$. The dimensionless magnetic field $\frac{B'}{B}$ is introduced where $B' = \frac{\hbar}{ea^2} = 0.0054T$ for $a = 350nm$. The strength of the electric modulation $V_0 = 0.2meV$ where as $B_0 = 0.004T$ which corresponds to $\omega_0 = 0.2meV$ with the result that both the modulations have equal strengths. In the same figure we have also shown the conductivity when either the magnetic or electric modulation alone is present. The $\frac{\pi}{2}$ phase difference in the bandwidths results in the same phase difference appearing in the conductivity for electric and magnetic modulations as can be seen in the figure. To better understand the effects of in-phase modulations on the conductivity we consider the asymptotic expression of the quantity ϕ given by Eq.(16) that appears in the magnetoconductivity σ_{yy} . The asymptotic results are valid when applied magnetic field is weak such that many Landau levels are filled. The asymptotic expression is obtained in the next section.

IV. ASYMPTOTIC EXPRESSIONS: IN-PHASE MODULATIONS

To get a better understanding of the results of the previous section we will consider the asymptotic expression of conductivity where analytic results in terms of elementary functions

can be obtained following [7, 10, 11]. The asymptotic expression of ϕ can be obtained by employing the following asymptotic expression for the Laguerre polynomials which is valid in the limit of large n when many Landau levels are filled [13]

$$\exp^{-u/2} L_n(u) \rightarrow \frac{1}{\sqrt{\pi \sqrt{nu}}} \cos(2\sqrt{nu} - \frac{\pi}{4}) \quad (17)$$

with the result that the in-phase bandwidth can be written as

$$\Delta(\text{in-phase}) = \frac{4\omega_0 \times \sqrt{\frac{2n}{u}} \times \sin(1/2\sqrt{\frac{u}{n}}) \times \sqrt{1+\delta^2}}{\sqrt{\pi \sqrt{nu}}} \times \sin\left(2\sqrt{nu} - \frac{\pi}{4} + \Phi\right)$$

where the ratio between the two modulation strengths $\delta = \frac{V_0 \cos(1/2\sqrt{\frac{u}{n}})}{2\omega_0 \sqrt{\frac{2n}{u}} \sin(1/2\sqrt{\frac{u}{n}})} = \tan(\Phi)$. The flat band condition from the above equation is $2\sqrt{nu} - \frac{\pi}{4} + \Phi = i\pi$ where i is an integer. This condition can also be expressed as $\frac{\sqrt{2n}}{a}l = i + \frac{1}{4} - \frac{\Phi}{\pi}$, where $n = n_F = \frac{\varepsilon_F^2}{\varepsilon^2(n)} = \frac{\varepsilon_F^2}{\hbar^2 \omega_g^2}$ is the highest Fermi integer. We see that the flat band condition in this case depends on the relative strength of the two modulations. We now take the continuum limit:

$$n \rightarrow \frac{1}{2} \left(\frac{l\varepsilon}{v\hbar} \right)^2, \sum_{n=0}^{\infty} \rightarrow \left(\frac{l}{v\hbar} \right)^2 \int_0^{\infty} \varepsilon d\varepsilon \quad (18)$$

to express ϕ in Eq.(16) as the following integral

$$\phi = \frac{4\omega_0^2 \times \frac{2\varepsilon_F^2}{u\hbar^2 \omega_g^2} \times \sin^2\left(1/2\sqrt{\frac{u\hbar^2 \omega_g^2}{\varepsilon_F^2}}\right) \times (1+\delta^2)}{\pi \sqrt{u}} \times \left(\frac{l}{v\hbar} \right)^2 \times \int_0^{\infty} \frac{\varepsilon}{\sqrt{n}} d\varepsilon \frac{g(\varepsilon)}{[g(\varepsilon)+1]^2} \sin^2(1/2\sqrt{u/n}) \sin^2(2\sqrt{nu} - \frac{\pi}{4} + \Phi) \quad (19)$$

where $u = 2\pi^2/b$ with $b = \frac{eBa^2}{\hbar} = \frac{B}{B'}$ and $B' = \frac{ea^2}{\hbar}$.

Now assuming that the temperature is low such that $\beta^{-1} \ll \varepsilon_F$ and replacing $\varepsilon = \varepsilon_F + s\beta^{-1}$, we rewrite the above integral as

$$\phi = \frac{4\omega_0^2 \times \frac{2\varepsilon_F^2}{u\hbar^2 \omega_g^2} \times \sin^2\left(1/2\sqrt{\frac{u\hbar^2 \omega_g^2}{\varepsilon_F^2}}\right) \times (1+\delta^2)}{\pi \sqrt{u}\beta} \times \left(\frac{l}{v\hbar} \right)^2 \times \int_{-\infty}^{\infty} \frac{ds e^s}{(e^s + 1)^2} \sin^2\left(\frac{2\pi p}{b} - \frac{\pi}{4} + \Phi + \frac{\sqrt{2u}}{v\sqrt{B\hbar}\beta} s\right) \quad (20)$$

where $p = \frac{\varepsilon_F a}{\hbar v} = k_F a = \sqrt{2\pi n_s} a$ is the dimensionless Fermi momentum of the electron. To obtain an analytic solution we have also replaced ε by ε_F in the above integral except in the sine term in the integrand.

The above expression can be written as

$$\phi = \frac{4\omega_0^2 \times \frac{2\varepsilon_F^2}{u\hbar^2\omega_g^2} \times \sin^2\left(\frac{1}{2}\sqrt{\frac{u\hbar^2\omega_g^2}{\varepsilon_F^2}}\right) \times (1 + \delta^2)}{\pi\sqrt{u}\beta} \times \left(\frac{l}{v\hbar}\right)^2 \times \int_{-\infty}^{\infty} \frac{ds}{\cosh^2(s/2)} \sin^2\left(\frac{2\pi p}{b} - \frac{\pi}{4} + \Phi + \frac{2\pi a}{vb\beta}s\right) \quad (21)$$

The above integration can be performed by using the following identity [13]:

$$\int_0^{\infty} dx \frac{\cos ax}{\cosh^2 \beta x} = \frac{a\pi}{2\beta^2 \sinh(a\pi/2\beta)} \quad (22)$$

with the result

$$\phi = \frac{2\omega_0^2 \times \frac{2\varepsilon_F^2}{u\hbar^2\omega_g^2}}{4\pi^2} \times \frac{T}{T_D} \times \sin^2\left(\frac{\pi}{p}\right) \times (1 + \delta^2) \times \left[1 - A\left(\frac{T}{T_D}\right) + 2A\left(\frac{T}{T_D}\right) \sin^2\left[2\pi\left(\frac{p}{b} - \frac{1}{8} + \frac{\Phi}{2\pi}\right)\right]\right] \quad (23)$$

where $k_B T_D = \frac{\hbar v b}{4\pi^2 a}$, $\frac{T}{T_D} = \frac{4\pi^2 a}{\hbar v b \beta}$ and $A(x) = \frac{x}{\sinh(x)} - (x \rightarrow -\infty) - \infty = 2xe^{-x}$.

From the asymptotic expression of ϕ given by Eq.(23), we observe that the effect of the in-phase electric and magnetic modulations is the appearance of a phase factor Φ in the conductivity. The shift in the Weiss oscillations when in-phase electric and magnetic modulations are present can be seen in Fig.(3). The phase factor Φ depends on the relative strength of the two modulations. How the Weiss oscillations are affected as Φ as well as the magnetic field is varied can be seen in Fig.(3). The results shown are for a fixed magnetic modulation of strength $\omega_0 = 0.2 meV$ and the electric modulation is varied. The change in V_0 results in a corresponding change in both δ and Φ . From Fig.(3), we observe that the position of the extrema in magnetoconductivity as a function of the inverse magnetic field depends on the relative strength of the modulations.

The effects of electric and magnetic modulations that are out-of-phase on the conductivity can be better appreciated if we consider the asymptotic expression for ϕ in this case. This is taken up in the next section.

V. MAGNETOCONDUCTIVITY WITH PERIODIC ELECTRIC AND MAGNETIC MODULATION: OUT-OF-PHASE

In this section, we calculate ϕ when electric and magnetic modulations are out of phase by $\pi/2$. We consider magnetic modulation out of phase with the electric one: We take the electric modulation to have the same phase as given in the previous section with the $\pi/2$ phase difference incorporated in the magnetic field. The energy eigenvalues are

$$\varepsilon(n, k_y) = \hbar\omega_g\sqrt{n} + \omega_0 \sin(Kx_0)G_n + V_0 F_n \cos(Kx_0), \quad (24)$$

and the bandwidth is

$$\Delta(\text{out of phase}) = \frac{4\omega_0 \times \sqrt{\frac{2n}{u}} \times \sin(1/2\sqrt{\frac{u}{n}})}{\sqrt{\pi\sqrt{nu}}} \times \sqrt{\delta^2 + (1 - \delta^2) \sin\left(2\sqrt{nu} - \frac{\pi}{4}\right)}. \quad (25)$$

The term responsible for Weiss oscillations is the $\sin(2\sqrt{nu} - \frac{\pi}{4})$ term under the square root which can be readily seen by considering the large n limit of the bandwidth. Therefore for $\delta = \pm 1$ Weiss oscillations are no longer present in the bandwidth. The velocity component v_y is given as

$$v_y = - \left[\frac{2\omega_0 u}{\hbar K} \cos(Kx_0)G_n(u) - \frac{2V_0 u}{\hbar K} \sin(Kx_0)F_n(u) \right] \quad (26)$$

and ϕ is given as

$$\phi = \sum_{n=0}^{\infty} \frac{g(\varepsilon_n)}{[g(\varepsilon_n) + 1]^2} \left[\frac{ue^{-u}V_0^2}{4} (L_n(u) + L_{n-1}(u))^2 + \omega_0^2 e^{-u} 2n(L_{n-1}(u) - L_n(u))^2 \right]. \quad (27)$$

The asymptotic expression for ϕ in the presence of both electric and magnetic modulations that are out of phase is obtained by substituting the asymptotic expressions for the Laguerre polynomials and converting the sum into integration with the result

$$\phi = \frac{2\omega_0^2 p^2}{\pi^2} \sin^2\left(\frac{\pi}{p}\right) \frac{T}{4\pi^2 T_D} \times \left[2\delta^2 + (1 - \delta^2) \left(1 - A\left(\frac{T}{T_D}\right) + 2A\left(\frac{T}{T_D}\right) \sin^2\left[2\pi\left(\frac{p}{b} - \frac{1}{8}\right)\right] \right) \right]. \quad (28)$$

From the expression of the out-of-phase bandwidth given by Eq.(25) we find that Weiss oscillations in the bandwidth are absent for relative modulation strength $\delta = \pm 1$, the same is reflected in ϕ as the term responsible for Weiss oscillations ($\sin^2[2\pi(\frac{p}{b} - \frac{1}{8})]$) vanishes for $\delta = \pm 1$ as can be seen from the above equation. Therefore the magnetoconductivity σ_{yy} does not exhibit Weiss oscillations when the relative modulation strength $\delta = \pm 1$. The

conductivity as a function of magnetic field when the electric and magnetic modulations are out-of-phase is shown in Fig.(4). The results shown are for a fixed magnetic modulation of strength $\omega_0 = 0.2meV$ and the electric modulation V_0 is allowed to vary between positive and negative values. The other parameters are the same as in Figs.(1,2,3). As V_0 is varied there is a corresponding change in δ . We find that the positions of the extrema of σ_{yy} as a function of the inverse magnetic field do not change as δ is varied since the phase factor Φ does not appear in the expression of conductivity when the two modulations are out of phase. It is also observed in Fig.(4) that there is a $\frac{\pi}{2}$ phase difference between the curves for $\delta \geq 1$ and $\delta < 1$. The same behavior is observed in the bandwidth which is reflected in the magnetoconductivity. In this work we have considered Weiss oscillations and have not taken pure Shubnikov de Hass (SdH) oscillations into account but SdH oscillations do appear superimposed on Weiss oscillations in the region of strong magnetic field as can be seen in all of our figures. In addition, since this work was motivated by [7], it is important to compare the results obtained here for graphene monolayer with those presented for standard 2DEG. In contrast to the work presented here, in [7] SdH are explicitly taken into account but we find that our principal expressions Eq.(23), (28) reduce to the expressions there when the terms contributing to SdH oscillations are ignored.

In the end, we would also like to mention relevance of this work in experimental studies of transport in graphene. Experiments on graphene monolayer in the presence of modulated electric and magnetic fields have not been realized yet, this theoretical work is in anticipation of experimental work. We expect that modulation effects predicted here can be observed in graphene employing established techniques that were used for the two-dimensional electron gas systems found in semiconductor heterostructures [14]. In conventional semiconductor systems, modulation of the potential seen by electrons can be produced by molecular beam epitaxy, chemical vapor deposition as well as sputtering techniques [15]. In graphene, we expect that modulation effects can be introduced by adsorbing adatoms on graphene surface using similar techniques, by positioning and aligning impurities with scanning tunneling microscopy or by applying top gates to graphene. Epitaxial growth of graphene on a patterned substrate is also possible [16].

In conclusion, we have determined the effects of both the electric and magnetic modulations on the magnetoconductivity of a graphene monolayer. Appearance of Weiss oscillations in the magnetoconductivity σ_{yy} is the main focus of this work. These oscillations are affected

by the relative phase of the two modulations and position of the oscillations depends on the relative strength of the two modulations. We find complete suppression of Weiss oscillations for particular relative strength of the modulations when the modulations are out-of-phase.

VI. ACKNOWLEDGEMENTS

We gratefully acknowledge F. M. Peeters for suggesting the problem and for comments on the manuscript. One of us (K.S.) would like to acknowledge the support of the Pakistan Science Foundation (PSF) through project No. C-QU/Phys (129). M. T. would like to acknowledge the support of the Pakistan Higher Education Commission (HEC).

*Department of Physics, Blackett Laboratory, Imperial College London, South Kensington Campus, London SW7 2AZ, United Kingdom.; Electronic address: m.tahir06@imperial.ac.uk

†Electronic address: ksabeeh@qau.edu.pk; kashifsabeeh@hotmail.com

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